Deep Problems for Bayesianism

David James Anderson

In his celebrated work, *An Enquiry Into Human Understanding*, David Hume produces a persuasive skeptical argument against inductive reasoning based on experience. Coupled with Rene Descartes' *Meditations on First Philosophy*, Hume's argument presents a grand challenge to modern epistemologists. Indeed, "challenge" accurately characterizes the true intent of Hume's seminal work. Hume does not claim to prove definitively that inductive reasoning is not justified; rather, he demonstrates the difficulty of making arguments for induction. Truly, Hume admits that it is "a man guilty of unpardonable arrogance who concludes, because an argument has escaped his own investigations, that therefore it does not really exist" (25).

Bayesianism purports to be this argument that has escaped Hume's investigations. By showing that our beliefs are consistent only if they adhere to a probability calculus, Bayesianism provides a justified model for induction based on experience. Unfortunately, when faced with Hume's exact skeptical challenge, the Bayesian model runs into some theoretical difficulties. What's more, the Humean skeptical challenge actually exploits some deeper problems with Bayesianism. In short, because Bayesianism acts like a process of elimination with respect to certain hypotheses, Bayesianism is inherently unqualified to model inductive reasoning of any sort.

To begin, we will briefly sketch Hume's skeptical challenge. Next, we will look at the philosophical foundations of Bayesianism. Third, we will examine exactly how Bayesianism purports to justify inductive hypotheses—its successes and shortcomings. Finally, we will see exactly how the Humean challenge reveals the deeper problems of Bayesianism and how a Bayesian might respond to these problems.

Hume's Skeptical Challenge

Hume suggests that if a hypothesis is confirmed many times in one's experience, it does not necessitate that it will be true in the future. Upon the consistent confirmation of a hypothesis in one's experience, it is natural to assume that such a hypothesis will be confirmed in the future. Truly, this sort of inductive reasoning is crucial both for science and our day-to-day survival. Hume calls into question this reasoning by showing that it rests on an unsupported assumption: the future resembles the past. Truly inductive reasoning often takes the following form:

(1) I have found the hypothesis h to be true in all past instances.

(2) The future resembles the past.

(3) The hypothesis h will be true in the future.

Such an inductive argument is valid but not sound because one cannot justify (2) without falling into a vicious circle.

As such, there is a challenge implicit in Hume's skepticism that is of interest to the Bayesian. Since Hume does not think we are justified in believing that the past resembles the future, clearly a justification of inductive reasoning must warrant our inductive conclusions without implicitly assuming that the future resembles the past. As such, it is Hume's skeptical challenge to show that the hypothesis

h_{100}, that a statement S will always be true

is more justified than the hypothesis

h_t, that the statement S is only true before time t.

Accordingly, by showing that h_{100} is more justified then h_t, we can justify inductive reasoning without necessarily assuming that the past resembles the future.

The Philosophic Foundations of Bayesianism

As a system of justification, Bayesianism attempts to justify the supporting connections between our beliefs by showing that these connections preserve consistency. The Bayesian uses an argument from probability and betting theory to show that our belief formation must follow certain rules to be consistent. Thus, we can justify the connections between our beliefs by showing that they follow the Bayesian rules and, thus, are consistent. Put simply, the Bayesian argument works as follows:

(1) Our degrees of belief can map to subjective probabilities.

(2) Our subjective probabilities imply a system of odds.

(3) Certain systems of odds are subject to a Dutch Book.

(4) Odds subject to a Dutch Book are inconsistent.

(5) Certain systems of beliefs are inconsistent by syllogism of 1-4.

(6) If we form beliefs according to the axioms of probability calculus, our system of beliefs will never be "Dutch

Book" inconsistent.

(7) Thus, we can justify connections between beliefs by the theorems based on the axioms of the probability calculus.

In all Bayesianism does not claim to address the regress argument and provide an ultimate foundation for belief; rather, it seeks to explain how we can justify the connections within a group of beliefs.

Observe that our beliefs in certain propositions come in degrees of certainty that we can map to numbers—subjective probabilities. Certainly, we are more confident in some beliefs than others. For example, although I might be absolutely certain that there is a pen on the desk in front of me, I may be only somewhat confident that my date will be at lunch on time. Clearly, we naturally prescribe different levels of certainty to our beliefs. Intuitively, these degrees of certainty can map onto an arbitrary range of numbers. As in the example above, my belief in the pen would map onto a number larger than my belief in my date's timeliness. For the Bayesian, certainty is a *subjective probability* the strength of which we measure by assigning a numerical value. One's subjective probability P in a belief h is expressed as the function $P(h)$. In all a subjective probability $P(h)$ is simply a measure of one's confidence in the hypothesis h.

As such, we believe that h in the sense required for knowledge when our subjective probability that h is sufficiently high. To know that h, one must truly believe, truly be certain that h. Indeed, according to the "true, justified belief" theory of knowledge, I only know that h if I believe with some level of confidence in the truth of h, and h is in reality true. My level of confidence in a belief is measured by my subjective probability; thus, if my subjective probability in h is sufficiently high, I believe that h. Determining the exact threshold at which simple beliefs become true conviction of the sort necessary for knowledge will not be essential for the Bayesian argument. Rather, it is enough to show that only a sufficiently high subjective probability that h implies that one truly believes that h.

To divert briefly from the topic, objective probabilities (as opposed to subjective probabilities) can describe "fair" betting odds. In betting, first the bookmaker offers betting odds $p{:}b$ against a hypothesis h and puts the sum b into the pot. Next the punter, who has some level of confidence in h, puts a sum p into the pot. Now the truth-value of h is revealed: if h, then the punter receives the pot; if $\sim h$, the bookmaker receives the pot. In such a scenario, given a certain real-world probability that h, the objectively fair odds are odds that confer no advantage or disadvantage to the punter or bookmaker based on this probability. For example, if the probability that h is 1/6, then the fair odds against h are 1:5 because larger rewards make up for the relatively slim chances for the punter. In other words, if the bookmaker and punter were to continue betting on h at these odds infinitely, neither person would, on average, make any money. As such, given an objective probability $P(h)$, we can always find the objectively fair betting odds $P(h):1-P(h)$ such that neither person is at an advantage.

Similarly, subjective probabilities can describe subjectively fair betting odds. If I believe that h with a certain probability $P(h)$, then I should I be inclined to accept a wager at or above the "fair" betting odds for that probability. Certainly, financial considerations or personal aversion to gambling may prevent me from ever taking such a "fair" bet; however, if I have this subjective probability, then I am at the very least deeply inclined to believe that such odds will confer no advantage to either side. Importantly, since these are subjectively fair betting odds, these odds may not *in fact* be fair, but are subjectively fair to the person who holds the subjective probability. In all, just like subjective probabilities numerically represent our confidence in a proposition, they similarly represent not so much an evaluation, but a deep feeling that tends to produce these odds.

If our beliefs can be represented as subjective probabilities which, in turn, can be represented as betting odds, then our system of beliefs is equivalent to a system of betting odds. Indeed, there is an intuitive appeal that "to possess a degree of belief, $P(h)$, in h is actually to be prepared to bet indifferently on or against h at odds $P(h):1-P(h)$" (Howson and Urbach, 91). In fact, we make bets like this all the time. For example, suppose I am driving home late at night. I have a certain level of confidence that I will get home safely; in other words, I have some subjective probability that I will get home safely. This may be different from the actual probability that I will get home safely, but it certainly is the probability that influences my decisions. In considering whether I should speed, I clearly weigh the reward (getting home earlier) with the penalty (death on 101). Certainly, the penalty outweighs the reward, but I might still be inclined to speed if my subjective probability of getting home safely is sufficiently high. Thus, the rewards and penalties for speeding form a sort of odds that I choose to accept based on my confidence in my driving ability. Intuitively, the connection between a system of betting odds and a system of belief is clear. But if our beliefs are analogous to a system of betting odds, what does this tell us abot the nature of our beliefs?

We begin by looking at a method for judging the fairness of a system of betting odds—a method that involves the Dutch Book. As explained above, we say the betting odds for a particular hypothesis are fair if they confer no advantage to either party. Similarly, we define a system of betting odds as fair if neither party can gain an advantage from exploiting this system. By convention, we say that any system that is ripe to be exploited in such a manner is subject to a "Dutch Book." Put simply, a Dutch Book is a system of stakes that, given some wagers, can ensure a net loss for the punter—regardless of the truth-values of the hypotheses.

For example, if your fair odds for the mutually exclusive hypotheses *a*, *b*, and *(a v b)* are 1:2. 1:2, and 1:1 respectively, then your system of betting odds is subject to a Dutch Book. Given these odds, should the bookmaker ask for a bet of 2 for *a*, 2 for *b*, and 3 against *(a v b)* at these odds you will accept. However, given any possible truth-value for *a* and *b*, you will always have a net loss. We are led to conclude, even if the odds for each different hypothesis are fair, the system of odds as a whole cannot be fair. In other words, while the odds 1:2 for *a* and *b* may be fair, the odds 1:1 for *(a v b)* cannot be fair. Given this example, if our system of beliefs acts like a system of odds, then what does it mean if this system of beliefs is subject to a Dutch Book?

Example of a Dutch Book against the betting odds a→1:2 b→1:2 and (a v b)→1:1

Possibilities	Your Bet	**Gain**	Loss	**Net Profit**
a and ~b	2	4	5	-1
~a and b	2	4	5	-1
~a and ~b	3	3	4	-1

The Bayesian makes a strong case that a system of beliefs subject to a Dutch Book is incoherent or at least inconsistent. Using the previous example, the Bayesian argues that there is something inconsistent in believing $P(a) = P(b) = 1/3$ and also believing $P(a \vee b) = 1/2$. Indeed, this argument has intuitive appeal. Suppose I am about ½ certain that David Hume had red hair, and ½ certain that he had brown hair. It seems, if my beliefs are consistent, that I should be quite certain that he had either red or brown hair. The Dutch Book, the Bayesian argues, tests for this sort of consistency in the connections between beliefs. Further, according to Bayesianism, to be inconsistent is to be unjustifiable. Inconsistent systems of beliefs are simply irrational. As such, the Bayesian looks for a way to create systems of beliefs that avoid the Dutch Book and, thus, are justifiable and consistent.

If your beliefs adhere to four axioms of probability calculus, then your beliefs cannot be exploited like a Dutch Book and will always be consistent. Avoiding the inconsistency of Dutch Books puts a certain set of constraints on any set of betting odds. It follows that since our subjective betting odds map directly onto a set of subjective probabilities, our formation of subjective probabilities is also constrained by the Dutch Book. According to the Bayesian, these constraints are expressed in four axioms of probability calculus. The Bayesian proves that if one does not obey the axioms in forming beliefs, then the resultant subjective betting odds will be subject to a Dutch Book. In short, if your degrees of belief are measured by subjective probabilities, then "consistency demands that they satisfy the probability axioms" (79). It immediately follows that for your belief in *h* to be justified with respect to your system of beliefs, then it must satisfy the axioms of probability calculus with respect to your other beliefs.

Consider an example concerning perception. Suppose I "know" the proposition *d* that there is a desk in front of me. When asked how I know that *d*, I might say that it is supported by a variety of other beliefs like "I am fairly certain that my vision is reliable," "I am certain that tables have this shape," "I am confident that there was a table here a minute ago," "I am somewhat certain that the table did not move." Given these other beliefs, one still might ask how these beliefs (if they themselves are justified) justify *d*. In response, I can show mathematically that *d* is the only consistent hypothesis given my other beliefs and the axioms of Bayesianism. In other words, my knowledge that *d* is consistent with and justifiable by my set of other beliefs by the laws of Bayesianism.

Having laid down the philosophic foundations of Bayesianism, we turn now to a specific kind of connection between beliefs: induction.

The Bayesian Justification of Induction—Virtues and Vices

Although Bayesianism claims to justify the connections between our beliefs, experiential induction seems to cause problems for the Bayesian picture. In the previous section, we saw how Bayesianism uses the Dutch Book to justify the connections between our beliefs. If Bayesianism truly justifies the connections between our beliefs, then it should be able to justify the connections between our belief in experience and conclusions we draw from that experience. In other words, Bayesianism should be able to justify induction. Nevertheless, while the Bayesian axioms provide a compelling account of experiential induction, they seem unable to refute the direct challenge of Hume's skeptical argument. Our exploration of the Bayesian picture of induction begins with Bayes's Theorem.

Bayes's Theorem provides a justified account of certain conditional probabilities. From the axioms of probability calculus, we can derive Bayes's Theorem: $P(h \mid e) = P(e \mid h)P(h) / P(e)$. In plain English, "The probability of the hypothesis given the event is equal to the probability of the event given the hypothesis times the probability of the hypothesis over the probability of the event." An addendum to Bayes's Theorem, the subjective probability that *e*, $P(e)$, has a useful equivalent expression based on the Total Probability Theorem. Given a subjective probability $P(e)$ and a set of hypothesis $h_1, h_2, h_3 \ldots h_n$ that are mutually exclusive and exhaustive, the Total Probability Theorem states that the probability that $P(e) = P(h_1)P(e \mid h_1) + P(h_2)P(e \mid h_2) + \ldots + P(h_n)P(e \mid h_n)$. Most important, by using substitution of the Total Probability Theorem into Bayes's Theorem, we get an alternate, and ultimately the most useful, construction of the Bayes's Theorem: $P(h \mid e) = P(e \mid h)\, P(h) / sum(\, P(h_n)P(e \mid h_n)\,)$.

From Bayes's Theorem, the Bayesian claims to have a rule for updating subjective probabilities based on experience—Bayesian conditionalization. Suppose $P(h)$ is your subjective probability before experiencing *e* and $P'(h)$ is your subjective probability after experiencing *e*. According to Bayesianism we should set $P'(h) = P(h \mid e)$. This stipulation seems natural enough; indeed, if $P(h \mid e)$ is your subjective probability that *h* given the event *e* then it follows that after the event *e* occurs, you should set $P'(h)$ to $P(h \mid e)$ to be consistent. Interestingly, there is some debate over whether setting $P'(h) = P(h \mid e)$ is actually Dutch Book justified; however, for the purposes of this paper, we will assume this assignment is justified. In all, Bayes's rule clearly gives us a model for updating subjective probabilities based on experience—induction.

For example, consider the belief that bread is nourishing. Consider three hypotheses:

h_0: No bread is nourishing

h_{50}: 50% of bread is nourishing

h_{100}: All bread is nourishing

Assume that these hypotheses are exhaustive of the hypotheses we are considering. Also, note that they are mutually exclusive. Before tasting any bread, we might favor one hypothesis over the other; in any case, our subjective probabilities for all three hypotheses need to add up to one because this is an exhaustive set of hypotheses. The issue of assigning subjective probabilities has been much debated in Bayesian literature; however, we will just assume indifference, assigning $P(h_n) = 1/3$. According to Bayes's Rule, as we begin to taste pieces of bread, our degrees of belief in each hypothesis should change in such a way that our beliefs are consistent and mirror our own process of inductive reasoning. Indeed, after the first taste of nourishing bread, e_1, our subjective probabilities change in a natural way: $P'(h_0) = 0$, $P'(h_{50}) = 1/3$, $P'(h_{100}) = 2/3$. As additional confirming experience is gathered, our inductive conclusion that all bread is nourishing continues to be justified by the Bayesian probability calculus. After four nourishing pieces of bread our subjective probability $P(h_{100}) = 16/17$ is compared to $P(h_{50}) = 1/17$.

At first glance, Bayesianism accounts for our inductive reasoning in a realistic manner; moreover, it justifies this reasoning by showing that these inductive leaps create a consistent system of belief. Clearly, the example shows how confirming experience with bread bolsters our hypothesis "all bread is nourishing." Indeed, where Hume believed we were unjustified in making assumptions based on experience, this Bayesian example shows that any belief system that *doesn't* respond to experience is inherently inconsistent and unjustified (except, of course, when the initial probability P(h) is 0).

Nevertheless, although the Bayesian seems to refute Hume's skeptical argument, the Bayesian model of induction fails to meet the exact skeptical challenge. In fact, the Bayesian system cannot prove that we are any more justified in believing some hypothesis will be true for all time than that a hypothesis will be true up until a certain date. To explore this possible Humean counter-example to the Bayesian argument, we look at a similar example.

Consider again the belief that bread is nourishing. We begin by considering the same three mutually exclusive and exhaustive hypotheses, plus a fourth: h_t "Bread is nourishing up until June 1." After the first taste of nourishing bread before June, e_1, our subjective probabilities change as follows: $P'(h_0) = 0$, $P'(h_{50}) = 1/5$, $P'(h_t) = 2/5$, $P'(h_{100}) = 2/5$. As additional confirming experience is gathered, $P(h_t)$ continues to equal $P(h_{100})$.

$P(h_{100})$ changes by the same amount as $P(h_t)$ after each consecutive confirming experience. In the preceding example, the hypothesis "all bread is nourishing" and the hypothesis "all bread is nourishing until June" remained equally subjectively probable after each confirming experience. Certainly, if we set the priors differently, we could artificially avoid this equality. Nevertheless, even with different priors, $P(h_{100})$ would still change by a similar amount as $P(h_t)$ after each consecutive confirming experience. As proof, after each experience we will always multiply both $P(h_{100})$ and $P(h_t)$ by the same quantity, $1/P(e)$. In other words, instead of the Bayesian theories' changing the values of $P(h_{100})$ and $P(h_t)$ to support $P(h_{100})$ and refute Hume, only the arbitrary value of the priors influences the outcome.

Clearly, to refute Hume's skeptical argument, Bayesianism needs to justify our being more inclined to believe $P(h_{100})$ than $P(h_t)$. Hume's skeptical argument claimed that we were no more justified in believing that a hypothesis will be true for all time than we are justified in believing that a hypothesis will be true up until some date. However, in everyday experience we think the former hypothesis is much more justified. Thus, to refute Hume's skeptical argument Bayesianism needs to show that $P(h_{100})$ is more justified than $P(h_t)$ after confirming evidence. Unfortunately for the Bayesian, the above example shows that Bayesianism does not provide this evidence and does not meet the skeptical challenge.

The Humean Counter-Example: A Larger Bayesian Problem

Just like the Humean counter-example shows that Bayesianism cannot justify induction over time, we can make a counter-example against induction over other traits. Consider the hypothesis h_a that "all tennis balls are bouncy" and h_b "all tennis balls except orange tennis balls are bouncy." We test tennis balls of all different shapes and sizes and they are all bouncy. In fact, just to be safe, we test tennis balls of over a thousand different colors. They too are bouncy. However, up until today we have not actually tested any orange tennis balls. Clearly, after testing tennis balls of a thousand different colors, we know that color has nothing to do with a tennis ball's bounce. As such, we would like to say that we know h_b is false, or at least very subjectively improbable. More to the point, we would like to *induce* that orange tennis balls are bouncy. Nevertheless, by Bayesian conditionalization, each hypothesis is equally probable—$P(h_a) = P(h_b)$. Just like in the Humean counter-example, Bayesianism does not justify induction. In light of this counter-example, Bayesianism is clearly incapable of justifying a variety of different types of induction—the Humean counter-example is not a "special case." So how do these counter-examples work?

In general, Humean-style counter-examples work by creating a hypothesis that accords with the inductive hypothesis *except* with respect to the inductive leap. In the tennis ball example, we attempt to make an inductive leap—that all tennis balls are bouncy, even though we have not tested orange tennis balls. The counter-example works by identifying this inductive leap and proposing a hypothesis that accords with this induction except with respect to orange tennis balls. As such, all evidence that supports the inductive hypothesis also supports the counter-hypothesis, h_b; thus, the hypotheses are equally justified. The Humean counter-example works in the same way. The inductive leap is to assume that because bread is nourishing in the past, it will also be nourishing in the future. The counter-hypothesis accords with this inductive hypothesis, except with respect to the future. Once again, all available evidence equally supports both hypotheses; thus, Bayesianism fails to justify induction. In all, it seems that given almost any inductive hypothesis we can create a counter-hypothesis that exploits this inductive leap and shows induction is unjustified.

In fact, the way the Humean counter-examples work exposes a larger problem with the Bayesian system. In short, with respect to certain types of hypotheses Bayesianism acts more like a sophisticated process of elimination than a model of human inductive reasoning. To begin, we distinguish a certain type of hypothesis: "non-probabilistic hypotheses." Such a distinction between probabilistic and non-probabilistic hypotheses is a real, meaningful distinction—all hypotheses behaving in the manner described by their category.

$P(e \mid h)$, where h is a "non-probabilistic" hypothesis, can only equal either 1, 0, or $P(e)$ given any event e. The hypothesis h_{100} in the Humean counter-example is a good example of a non-probabilistic hypothesis. Given e_n, a nourishing piece of bread, $P(e_n \mid h_{100})$ will always be 1. Similarly, given e_p, a non-nourishing piece of bread, $P(e_p \mid h_{100})$ will always be 0. $P(e \mid h_{100}) = P(e)$ only in the case where e and h_{100} are probabilistically independent—they have nothing to do with each other. For example, if e_i signifies "daisies growing in the garden," the conditional probability *of* $P(e_i \mid h_{100}) = P(e_i)$. Clearly, by the axioms of probability $P(e_i \mid h_{100}) = P(e_i \wedge h_{100}) / P(h_{100})$. Further, because when a and b are probabilistically independent, $P(a \wedge b) = P(a)P(b)$, it follows that $P(e_i \mid h_{100}) = P(e_i)P(h_{100})/P(h_{100}) = P(e_i)$. Most importantly, h_{100} represents a non-probabilistic hypothesis because $P(e \mid h_{100})$ cannot equal anything besides 1, 0, or $P(e)$ *given any event e*. More to the point, $P(e \mid h_{100})$ can never equal .5 or .3 unless, of course, e and h_{100} are probabilistically independent. So how do non-probabilistic hypotheses behave in the Bayesian system?

Given a set of mutually exclusive non-probabilistic hypotheses, one's subjective probabilities in these hypotheses can only (1) remain unchanged, (2) rise at a constant rate across all hypotheses, or (3) go to zero based on a single event. First, one's subject probabilities in a non-probabilistic hypothesis remain effectively unchanged when all events are probabilistically independent of the hypothesis. In such a scenario, $P(e \mid h) = P(e)$. When this result is applied to Bayesian conditionalization, $P'(h) = P(h)$. Second, one's subjective probabilities rise at a constant rate across all hypotheses when all events accord with the hypotheses. For example, consider two non-probabilistic hypotheses: h_r all red balls are bouncy and h_a all balls are bouncy. Every time we experience e_r, a red bouncy ball, $P(e_r \mid h_r) = P(e_r \mid h_a) = 1$. Thus, our subjective probabilities of $P(h_r)$ and $P(h_a)$ rise at the same rate. Finally, one's subjective probabilities in a non-probabilistic hypothesis can go to zero given a single event. In the case of h_{100}, all bread is nourishing, eating just one piece of non-nourishing bread, e_p, fully eliminates this hypothesis. $P(e_p \mid h_{100}) = 0$ so $P'(h_{100}) = 0$.

The unfortunate implication of these observations is that, with respect to non-probabilistic hypotheses, Bayesianism behaves like a sophisticated process of elimination. Consider a set of non-probabilistic hypotheses $(h_1 \ldots h_{10})$ with equal prior probabilities $(P(h_n) = 1/10)$. Inevitably I experience events that affect my subjective probabilities in these hypotheses. If the event is in accordance with some of my hypotheses $h_1 \ldots h_9$, then I increment my subjective probability in these hypotheses *equally* across $h_1 \ldots h_9$. If the event is counter to a hypothesis h_{10}, I eliminate the hypothesis and set $P(h_{10}) = 0$. Observe that this is essentially a process of elimination. As events come in, I either eliminate a hypothesis, leave its subjective probability unchanged, or adjust the subjective probability equally to all others. Indeed, I can never encounter an affirming event that will cause my subjective probability of $P(h_1)$ and $P(h_2)$ to rise at *different rates*. Instead, $P(h_1)$ and $P(h_2)$ either rise at the same rate, or I eliminate a hypothesis. In a process of elimination, all hypotheses remain relatively equally attractive that have not been proven otherwise by the evidence at hand. Clearly, with respect to non-probabilistic hypotheses, Bayesianism behaves similarly to a process of elimination.

A system capable of induction needs to support subjective probabilities that can change at *different rates* given the same evidence. As shown above, for Bayesianism to justify induction, it needs to show that an inductive hypothesis is justifiably more subjectively probable than the counter-inductive hypothesis. For instance, in the Humean counter-example, to justify induction, Bayesianism needs to show that $P(h_i) < P(h_{100})$ given the same evidence and the same priors. Truly, any system capable of supporting induction needs to support subjective probabilities growing at different rates given the same evidence.

Because a process of elimination does not support subjective probabilities changing at different rates given the same evidence, it seems that Bayesianism is inherently unqualified to justify induction of non-probabilistic hypotheses. As shown above, the Bayesian system operates like a sophisticated process of elimination with respect to non-probabilistic hypotheses. As such, Bayesian conditionalization will always grow subjective probabilities at the same rate, except to fully eliminate a hypothesis. Nonetheless, an inductive system needs to grow subjective probabilities at different rates—exactly what Bayesianism, as a process of elimination, can't do. In all, this deep relation between Bayesian conditionalization of non-probabilistic hypotheses and a process of elimination lies at the heart of Bayesianism's inability to justify induction.

One might object that even if the Bayesian handling of non-probabilistic hypotheses is inherently unqualified to handle induction, the Bayesian handling of probabilistic hypotheses does not suffer from the same problem. We defined non-probabilistic hypotheses as those hypotheses that when considered as condition probability $P(e \mid h)$ returned *1,0*, or $P(e)$ for any e. However, probabilistic hypotheses like "50% of bread is nourishing" or "only two pieces of bread are nourishing" can yield $P(e \mid h) = [0..1]$. In other words, probabilistic hypotheses can change at different rates given the same evidence. As such, it seems that the Bayesian handling of probabilistic hypotheses might not be so "inherently unqualified" to handle induction.

Although Bayesianism certainly does not act like a process of elimination with respect to probabilistic hypotheses, Bayesianism is still unsuited to justify induction of probabilistic hypotheses. Observe that probabilistic inductive hypotheses like h_{50}, "50% of bread is nourishing," all have an inductive leap. For h_{50}, the leap is the assumption that because 50% of bread was nourishing in the past it will continue to be so in the future. Using the same method described above, we can formulate a hypothesis that exploits this inductive leap—h_i, 50% of bread will be nourishing until tomorrow when no bread will be nourishing. Once again, the subjective probabilities of the two hypotheses change at the same rate given the same evidence. In all, certainly the subjective probabilities of probabilistic hypotheses do not necessarily change at the same rate given the same evidence (like they do with non-probabilistic hypotheses); nevertheless, we can still construct two hypotheses that do, in fact, change at the same rate given the same evidence up to a point. Thus, even probabilistic hypotheses are subject to Humean counter-examples to induction, though not for entirely the same reasons.

One might also object that non-probabilistic hypotheses can, in fact, be construed to change at different rates. Certainly, when e signifies eating a nourishing piece of bread $P(e \mid h_{100}) = 1$, because given that all bread is nourishing, it follows that every experience of eating bread will be nourishing. Similarly, $P(e \mid h_t)$ also equals one because assuming it is before June all bread must be nourishing. However, perhaps we can interpret $P(e \mid h_t)$ such that it does not equal one. If we consider e not as "eating nourishing bread right now" but as "eating nourishing bread in general" then $P(e \mid h_t)$ is certainly less than 1. Consider that h_t means that only the bread before June will be nourishing. Further, I will experience eating bread both before and after June. Thus, "the probability that bread will be nourishing given the hypothesis" might be interpreted as "the probability that my experience of eating bread will be before June." Given that I will have experiences of eating bread before and after June (in fact, my next experience might be after June), this probability will certainly be less than 1. Most importantly, if $P(e \mid h_t) < 1$ then $P(h_t)$ and $P(h_{100})$ change at different rates—supporting induction.

Nevertheless, whether or not this interpretation of $P(e \mid h_t)$ makes sense, the inconsistencies this interpretation raises in the overall Bayesian system show that $P(e \mid h_t)$ cannot be less than one. If we consistently interpret e as "the general experience of eating bread," we are bound to have inconsistent beliefs. Suppose we eat a piece of non-nourishing bread before June. With this interpretation of e as a general experience, we are committed to believing that h_t even though h_t is definitely false. In short, such an interpretation inevitably leads us to a situation where we have a $P(h_t)$ that is above 0 even though we have eaten non-nourishing bread before June.

In concluding, certainly Bayesian conditionalization provides a compelling story of how our beliefs can be justified given certain relevant experiences. Nevertheless, because Bayesian conditionalization of non-probabilistic hypotheses behaves like a process of elimination, it is inherently unqualified to justify inductive logic. In all, by identifying the deep Bayesian problems that give Humean counter-examples their force, we have defined more precisely the task ahead for the defender of justified induction.

Stanford University

REFERENCES

Descartes, Rene. *Discourse on Method and Meditations on First Philosophy*. Donald Cress, trans. Indianapolis: Hackett, 1998.

Earman, John. *Bayes or Bust?: A Critical Examination of Bayesian Confirmation Theory*. Cambridge, Mass: MIT Press, 1992.

Howson, Colin, and Peter Urbach. *Scientific Reasoning: The Bayesian Approach*. Los Angeles: Salle Open Court, 1989.

Hume, David. *An Enquiry Concerning Human Understanding*. Indianapolis: Hackett, 1998.